Published by Institute of Physics Publishing for SISSA/ISAS
Received: *April 25, 2002*
Accepted: *June 24, 2002*

Nonplanar anomalies in noncommutative theories and the Green-Schwarz mechanism

Adi Armoni

Theory Division, CERN
CH-1211 Geneva 23, Switzerland
E-mail: adi.armoni@cern.ch

Esperanza Lopez and Stefan Theisen

Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut
Am Mühlenberg 1,D-14476 Golm, Germany
E-mail: lopez@aei-potsdam.mpg.de, theisen@aei-potsdam.mpg.de

Abstract: We discuss nonplanar anomalies in noncommutative gauge theories. In particular we show that a nonplanar anomaly exists when the external noncommutative momentum is zero and that it leads to a non-conservation of the associated axial charge. In the case of nonplanar local anomalies, a cancellation of the anomaly can be achieved by a Green-Schwarz mechanism. In an example of D3 branes placed on an orbifold singularity that leads to a chiral theory, the mechanism involves twisted RR fields which propagate with zero noncommutative momentum. Global anomalies are not cancelled and, in particular, the decay $\pi^0 \to 2\gamma$ is allowed.

Keywords: Anomalies in Field and String Theories, Non-Commutative Geometry.

JHEP06(2002)050

http://jhep.sissa.it/archive/papers/jhep062002050/jhep062002050.pdf

Contents

1. Introduction

Anomalies play a fundamental role in particle physics [1]. In field theory, local anomalies signal an inconsistency of the theory and are therefore considered as 'bad'. Global anomalies are 'good' since they allow decay processes, such as $\pi^0 \to 2\gamma$, which are classically forbidden but observed in nature. Through anomaly matching conditions, global anomalies restrict microscopic and effective descriptions [2] which was important in establishing Seiberg duality [3]. In a consistent string theory, all (local) anomalies must cancel. The manifestation of the consistency in the low energy effective theory is, however, subtle, as it might involve a cancellation between a tree level diagram and a one loop diagram. This is the Green-Schwarz mechanism [4].

Recently, there has been much interest in noncommutative field theories, see [5, 6] for reviews. The issue of anomalies in the framework of noncommutative chiral gauge theories was already discussed by several authors [7]–[17]. There are two types of possible anomalies: planar and nonplanar. Planar anomalies are well understood. They result from graphs that 'behave' exactly as in the commutative theory apart from an overall phase factor which depends only on external momenta [18, 19], and therefore automatically obey the usual anomaly equations, such as e.g. eq. (2.5) below.

The issue of nonplanar anomalies is subtle. Nonplanar graphs are UV finite at one loop and therefore it was suggested that these anomalies automatically vanish [14, 15, 16].

However, Ardalan and Sadooghi pointed out [11] that due to UV/IR mixing [20], the nonplanar anomaly does not vanish (see also [17]).[1]

The goal of this work is to clarify the issue of nonplanar anomalies. On one hand, if one views noncommutativity as a (gauge invariant) regulator, it is hard to believe that one can introduce a regulator that leads to a conservation of the axial charge (noncommutativity is, however, not a good regulator in the sense that the limit $\theta \to 0$ is not smooth). Another argument against the vanishing of the anomaly arises from $\mathcal{N} = 2$ SYM [21]. In this theory the $\mathrm{U}(1)_{\mathcal{R}}$ anomaly belongs to the same multiplet of the conformal anomaly (the β-function). Since the β function, generically, does not vanish, it would be surprising if the $\mathrm{U}(1)_{\mathcal{R}}$ anomaly vanishes. On the other hand the paradox raised in [14] seems to suggest the vanishing of the nonplanar anomaly as a unique resolution.

The paradox is the following: consider a $\mathrm{U}(N)$ chiral theory which can be realized by a brane configuration in string theory. Suppose that the U(1) is potentially anomalous, due to an excess of left handed fermions over right handed fermions. In the ordinary theory, that would lead to an anomaly, which can be resolved in string theory by giving an infinite mass to the U(1), leaving at low energies an $\mathrm{SU}(N)$ theory and a global U(1). In those non-commutative gauge theories which have been obtained as effective low-energy field theories of string theory one always obtains $\mathrm{U}(N)$ gauge groups, rather than $\mathrm{SU}(N)$. In these theories the noncommutativity mixes the U(1) and the $\mathrm{SU}(N)$ parts and the above scenario cannot occur [22, 23]. A way out, which was proposed in [14] is that actually there is no anomaly, though the matter content is left-right asymmetric, due to the finiteness of the associated nonplanar graph.

We would like to present a different solution: the nonplanar anomaly graph is indeed finite and actually vanishes, for any *non-zero noncommutative external momentum*. However, the graph is not regulated for exactly zero external noncommutative momentum — which leads to the integrated anomaly equation

$$\int d^2x_{NC}\ \partial_\mu j_A^\mu = -\frac{g^2}{8\pi^2}\int d^2x_{NC}\ F_{\mu\nu}\tilde{F}^{\mu\nu}\,. \tag{1.1}$$

In this expression, the integration is restricted to the noncommutative plane.[2] In particular the anomaly does not vanish. As we shall see, despite of the anomaly, string theory avoids the inconsistency via a subtle Green-Schwarz mechanism, which involves closed strings with exactly zero noncommutative momentum. The resulting low-energy theory would be $\mathrm{U}(N)$ (with $\mathrm{U}(N)$ gauge invariance!), with the U(1) being massive only if it carries zero noncommutative momentum.

We will also discuss global anomalies. Here we claim that due to (1.1) the global anomaly does not vanish. In particular the π^0 decays. In addition, if one considers anomaly matching conditions, nonplanar anomalies should be taken into account as well.

[1]It is interesting to note that a similar phenomenon of vanishing anomaly exists for the lattice version of the noncommutative theory [13]. It is, however, not clear whether it survives the continuum limit.

[2]A similarly needed integration over the noncommutative plane was pointed out [24] in relation with renormalization scale independence of composite operators.

JHEP06(2002)050

The organization of the manuscript is as follows: in section 2 we describe the ambiguity (or freedom) in the definition of currents in the noncommutative theory and the relevance of non-planar anomalies. In section 3 we review the arguments, using perturbation theory, in favor of the vanishing of the anomaly for any $\theta q \neq 0$ and we show that the behavior at $\theta q = 0$ leads to an integral anomaly equation. In section 4 we show that the point splitting definition of the non-planar current gives rise to the same integrated anomaly equation. We carry the analysis both for two and four dimensions. In section 5 we discuss, briefly, mixed anomalies. Section 6 is devoted to a discussion and a resolution of the above mentioned paradox in field theory and in string theory. Finally, in section 7 we discuss the consequences of our findings on global anomalies.

We use the following conventions throughout the manuscript. $[x^\mu, x^\nu]_* = i\theta^{\mu\nu}$ and θq stands for $\theta^{\mu\nu} q_\nu$. In 4d we consider only theories with space-space noncommutativity and in 2d we assume an euclidean signature. Frequently we will use the notation $F(q) = f(q)|_{\theta q = 0}$. By this we mean that $F(q) = f(q)$ only if q has a zero component along the noncommutative directions, otherwise $F(q) = 0$.

2. Anomalies in noncommutative gauge theories

We will consider a non-commutative gauge theory with group U(1) and a massless Dirac fermion transforming in the fundamental representation. The fermionic action is

$$S = i \int d^d x\, \bar{\psi} * \not{D} \psi \,, \tag{2.1}$$

where $D_\mu \psi = \partial_\mu \psi + i g A_\mu * \psi$. This action is invariant under global axial transformations

$$\delta_\alpha \psi(x) = i\, \alpha\, \gamma^5 \psi(x) \,. \tag{2.2}$$

In trying to derive the axial associated current we are faced with a problem [8, 11]. If we define the axial current following the Noether procedure, we have to decide if the lagrangian is given by (i) $\mathcal{L} = i\bar{\psi} * \not{D}\psi$ or instead (ii) $\mathcal{L}' = -i(\not{D}\psi)^t * \bar{\psi}^t$.[3] The two lagrangians lead to the same action since under integration the $*$-product satisfies cyclic symmetry. Equivalently, in order to derive the axial current we can formally promote $\alpha \to \alpha(x)$. Then we should choose $\alpha(x)$ to multiply $\psi(x)$ on (i) the right or (ii) the left. The axial currents associated to these two choices are

$$\text{(i)}\ \ j_A^\mu = \bar{\psi} * \gamma^\mu \gamma^5 \psi \,, \qquad \text{(ii)}\ \ j_A'^{\,\mu} = -\psi^t (\gamma^\mu \gamma^5)^t * \bar{\psi}^t \,. \tag{2.3}$$

The conservation properties of these two currents have been extensively studied, giving different results for j_A and j_A' [7]–[16]. This apparently puzzling conclusion has the following origin. Although the lagrangians $\mathcal{L}$ and $\mathcal{L}'$ are just related by a total derivative, there is a crucial difference between them. While $\mathcal{L}$ is gauge invariant, $\mathcal{L}'$ behaves under gauge transformations as an operator in the adjoint representation. These transformation

[3] Of course one can imagine an intermediate definition interpolating among the two previous possibilities.

JHEP06(2002)050

properties are inherited by j_A and j'_A. At the classical level these currents are conserved and covariantly conserved respectively [8, 11]

$$\text{(i)}\ \ \partial_\mu j^\mu_A = 0\,, \qquad \text{(ii)}\ \ D_\mu {j'}^\mu_A = \partial_\mu j'^\mu_A + ig\,[A_\mu, {j'}^\mu_A]_* = 0\,. \tag{2.4}$$

At the quantum level, j'_A has been shown to satisfy the non-commutative counterpart of the ordinary anomaly equation [7, 8]. In four dimensions

$$D_\mu {j'}^\mu_A = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} * F_{\rho\sigma}\,. \tag{2.5}$$

The current j_A cannot satisfy a similar equation, since $F_{\mu\nu}$ is not gauge invariant. One could think a priori of two possibilities. The divergence of j_A is equal to some gauge invariant completion of $F \wedge F$ involving Wilson line operators. This however would imply that there is an infinite number of Feynman graphs contributing to the anomaly equation, in contrast to the ordinary case and to (2.5). The second possibility is that the divergence of j_A remains zero at the quantum level. Explicit calculations have shown that this is indeed the case [10], except for a subtlety. The previous argument does not constrain those components of the divergence with zero momentum along the non-commutative directions, since $F \wedge F|_{\theta q=0}$ is gauge invariant. Indeed, we will show that j_A satisfies

$$\int d^2x_{NC}\, \partial_\mu j^\mu_A = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\int d^2x_{NC}\, F_{\mu\nu}\, F_{\rho\sigma}\,. \tag{2.6}$$

The subindex NC implies integration along the non-commutative plane. The existence of a restricted anomaly affecting the current j_A was first pointed out in [11].

Although we have treated the simple case of the axial symmetry, the previous considerations apply to any global symmetry and to a U(N) gauge group. Let us analyze the implications of equation (2.6). We consider the case that $\theta^{0i} = 0$, i.e. non-commutativity restricted to space coordinates. If the cyclic symmetry of the $*$-product under integration holds, both currents define the same gauge invariant charge

$$Q = \int \mathbf{dx}\, j^0_A = \int \mathbf{dx}\, {j'}^0_A\,. \tag{2.7}$$

The anomaly equation (2.5) implies that Q is not conserved in the presence of non-zero instanton number. If at the same time the divergence of j_A did vanish, we would encounter a contradiction. This problem does not arise when j_A satisfies instead (2.6). Then, independently of which current we use, we will arrive at the same conclusion about the variation of the axial charge.

3. Perturbative non-planar anomaly calculation

3.1 Two dimensions

We want to derive the anomaly equation satisfied by the gauge invariant axial current j_A in two-dimensions. We work in euclidean space, since noncommutativity in the time coordinate leads to a non-unitary theory [25]. We need to evaluate $\langle j^\mu(q) j^\nu_A(-q)\rangle$, where

j^μ denotes the vector current, i.e. $j^\mu = \psi^t \gamma^{\mu t} * \bar{\psi}^t$ for a U(1) theory. It is convenient to use the relation $\gamma^\mu \gamma^5 = i\epsilon^{\mu\nu}\gamma_\nu$, valid in two dimensions. This allows us to concentrate on the correlator $\langle j^\mu(q) j'^{\,\nu}(-q)\rangle$, with $j'^{\,\mu} = \bar{\psi} * \gamma^\mu \psi$. A straightforward and simple calculation gives

$$\langle j^\mu(q) j'^{\,\nu}(-q)\rangle = -\int \frac{d^2 l}{(2\pi)^2}\,\mathrm{tr}\left\{\gamma^\mu \frac{\not{l}}{l^2}\gamma^\nu \frac{(\not{l}+\not{q})}{(l+q)^2} e^{il\cdot\theta\cdot q}\right\} \tag{3.1}$$

where the only difference compared to the commutative case is the appearance of the momentum-dependent phase factor. Due to the different order of the fields with respect to the $*$-product in j and j', the correlator of this two operators gives rise to a non-planar graph (see figure 1).

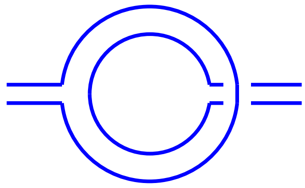

Figure 1: The non-planar anomaly in 2d.

Performing the traces, changing variables and using dimensional regularization in (3.1), one obtains

$$\langle j^\mu(q) j'^{\,\nu}(-q)\rangle = 2\int_0^1 dx \int \frac{d^d l}{(2\pi)^d}\,\frac{2l^\mu l^\nu - \delta^{\mu\nu} l^2 - 2x(1-x)q^\mu q^\nu + \delta^{\mu\nu} x(1-x) q^2}{(l^2 + x(1-x)q^2)^2} e^{il\cdot\theta\cdot q} \tag{3.2}$$

For non-vanishing non-commutative momentum $(\theta q)^2$, the loop-momentum dependent phase factor renders the integral UV finite and dimensional regularization is, in fact, not necessary. The integral can be expressed in terms of Bessel functions which add up to zero. Thus, for any non-zero θq, the r.h.s. of the previous equation is zero. However, at $(\theta q)^2 = 0$,[4] the non-commutativity parameter does not act as a regulator and we have to use dimensional regularization. The result for the integral is the same as in the commutative theory at zero momentum.

We conclude that j_A is conserved at all non-vanishing θq, i.e. $q_\mu j_A^\mu = 0$, and the non-conservation at $\theta q = 0$ can be expressed in the form

$$\int d^2x\, \partial_\mu j_A^\mu(x) = i\frac{g}{2\pi}\int d^2x\, \epsilon^{\mu\nu} F_{\mu\nu}(x)\,. \tag{3.3}$$

This is an integral equation for the 2d anomaly. Thus we learn that the anomaly exists and that it is concentrated in the zero momentum component of the current.

3.2 Four dimensions

We now repeat the previous argument for $d = 4$. Contrary to the $d = 2$ case, we will consider only space non-commutativity. We need to evaluate the triangle graph with one insertion of the axial current j_A. As before, the resulting diagram is non-planar (see figure 2).

In [10] the anomalies in a $\mathrm{U}(N_1)\times\mathrm{U}(N_2)$ non-commutative gauge theory with a chiral fermion in the bifundamental representation were treated in great detail. Their results imply that if the axial current is coupled through the non-planar vertex, then for any non-

[4] In two-dimensional euclidean space, $(\theta q)^2 = 0$ implies $q = 0$.

zero non-commutative momentum θq it satisfies $q_\mu j_A^\mu(q) = 0$. The reason is that for any $\theta q \neq 0$, due to loop momentum dependent phase factors, the integral no longer diverges linearly and the momentum variable can be shifted. In particular, the phase factors in the integrals which one gets from the two contributing Feynman diagrams coincide after the momentum shift and one obtains exact cancellation.

Again, as in the two-dimensional case, this argument breaks down for zero external non-commutative momentum, since in this case the dependence on θ vanishes and the integrals are not regulated by non-commutativity. The shift in the integral now yields a surface term which gives the anomaly. We then get the known result for the triangle anomaly for the commutative theory. In summary, the relation (2.6) holds independent of whether the axial current is coupled via the planar or non-planar vertex.

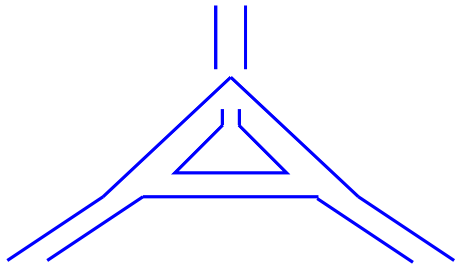

Figure 2: The non-planar anomaly in 4d.

In spite of this, it could have seemed tempting to *define* the value of the non-planar anomaly diagrams at zero θq as the limit from θq non-zero. This point of view was followed in [10, 14, 16], which led to the conclusion that non-planar anomalies cancel. We have argued that this way of defining correlators leads to inconsistencies, which are resolved if a non-planar anomaly survives at zero non-commutative momentum. This is a surprising conclusion from the perspective of ordinary quantum field theories, since it implies that the non-planar anomaly graphs radically violate the usual analyticity properties of Green functions. These properties are derived from the requirement that commutators of fields vanish for space-like separations. However in the non-commutative case this does not need to hold along the non-local, non-commutative directions. In particular, it has been shown that superluminal propagation is possible along the non-commutative directions [26, 27]. Therefore, there are no a priori requirements on the behaviour of quantum correlators as a function of the variable θq. On the contrary we should expect an ordinary behaviour on the variable $q^2_{\rm com} = q_0^2 - q_1^2$ [28]. Notice that we are considering space non-commutativity: $[x^2, x^3] = i\theta$.

It has been known since [20] that the limit $\theta \to 0$ of non-commutative Green functions is not smooth. A clear example of this is the existence of a non-zero, θ independent, beta-function for pure non-commutative U(1) [29, 23]. In this paper we are encountering similar discontinuities of correlators as functions of the non-commutative momentum. This could have been expected from the fact that θ only enters the Feymann integrals in the combination θq.

Without restricting to the particular case studied here, one might wonder what is the correct way to define the value of generic correlators at zero non-commutative momentum. It would be important to carefully study the consequences of a loss of analyticity. Another interesting question is the behaviour, with respect to this issue, of non-commutative field theories derived from string theory. Following this line of ideas, we will discuss in section 6 the non-commutative Green-Schwarz mechanism.

4. Non-planar anomaly via point splitting

The point splitting method provides an alternative and neat derivation of the non-planar anomaly. Composite operators containing product of fields evaluated at the same point are generically singular. A proper definition involves a regularization procedure, which can break some of the symmetries existing at the classical level. The current j_A, regulated by point splitting, is

$$j_A^\mu(x) = \lim_{\epsilon\to 0} \bar{\psi}\left(x+\frac{\epsilon}{2}\right)\gamma^\mu\gamma^5 * \mathcal{U}\left(x+\frac{\epsilon}{2}, x-\frac{\epsilon}{2}\right) * \psi\left(x-\frac{\epsilon}{2}\right), \tag{4.1}$$

where a Wilson line has been introduced to preserve the gauge invariance of the regularized expression,

$$\mathcal{U}(x,y) = e^{ig\int_x^y dl\cdot A(l)}\,. \tag{4.2}$$

The divergence of j_A^μ can be derived by expanding the Wilson line in (4.1) at linear order in ϵ and using the equations of motion of the fermionic fields [7]

$$\partial_\mu j_A^\mu(x) = -ig\lim_{\epsilon\to 0}\epsilon^\nu\bar{\psi}\left(x+\frac{\epsilon}{2}\right)\gamma^\mu\gamma^5 * F_{\mu\nu}(x) * \psi\left(x-\frac{\epsilon}{2}\right). \tag{4.3}$$

The r.h.s. of this equation, although proportional to ϵ, may give a finite result since the correlator of the fermion fields can become singular as the regulator, ϵ, is removed. Namely, in Fourier space we will have

$$\begin{aligned} iq_\mu j_A^\mu(q) &= -ig\lim_{\epsilon\to 0}\epsilon^\nu\int\frac{d^dl}{(2\pi)^d}\frac{d^dp}{(2\pi)^d}\, e^{\frac{i}{2}(l-p)\epsilon}e^{-\frac{i}{2}(l\theta q+q\theta p+p\theta l)}\times \\ &\qquad\times\langle\bar{\psi}(l)\,\gamma^\mu\gamma^5\psi(p)\rangle F_{\mu\nu}(q-l-p)\,, \end{aligned} \tag{4.4}$$

with d the space-time dimension. We will treat first the two-dimensional case.[5] Substituting the tree level fermion correlator and performing the p integration we obtain

$$\begin{aligned} iq_\mu j_A^\mu(q) &= ig\lim_{\epsilon\to 0}\epsilon^\nu F_{\mu\nu}(q)\,\mathrm{tr}(\gamma^\rho\gamma^\mu\gamma^5)\int\frac{d^2l}{(2\pi)^2}\frac{l_\rho}{l^2}e^{il(\epsilon-\theta q)} \\ &= i\frac{g}{\pi}\lim_{\epsilon\to 0}\frac{\epsilon^\nu(\epsilon-\theta q)_\rho}{(\epsilon-\theta q)^2}\,\epsilon^{\mu\rho}F_{\mu\nu}(q)\,. \end{aligned} \tag{4.5}$$

When $\theta q \neq 0$, the r.h.s. of this expression tends to zero. We recover thus the result that non-planar anomalies cancel. The order in which the fermion fields and the field strength are multiplied in (4.3) is at the origin of this cancellation. It causes that, even after sending ϵ to zero, the fermion fields are effectively separated by a distance θq, being q the momentum carried by the field strength. Therefore their correlator does not become singular. This makes clear why the cancellation is not operative at $\theta q = 0$. Indeed, at this value (4.5) gives a non-zero result, as it is the case for ordinary theories. In dimension d we have $\lim_{\epsilon\to 0}\frac{\epsilon^\nu\epsilon_\rho}{\epsilon^2} = \frac{\delta^\nu_\rho}{d}$. Substituting this in (4.5) we obtain that the current j_A^μ satisfies the integrated form of the standard anomaly, equation (3.3). This fact is implicit in the analysis of [7].

[5] Here we also assume an euclidean space.

The anomaly of the non-gauge invariant axial current j'_A in (2.3) can be studied in the same way. The regularized version of the current is in this case

$$j'^{\mu}_A(x) = -\mathcal{U}\left(x, x+\frac{\epsilon}{2}\right) * \psi^t\left(x+\frac{\epsilon}{2}\right)(\gamma^\mu\gamma^5)^t * \bar{\psi}^t\left(x-\frac{\epsilon}{2}\right) * \mathcal{U}\left(x-\frac{\epsilon}{2}, x\right). \tag{4.6}$$

The Wilson lines have been introduced in order that the point splitting regularization does not alter the behavior of the current under gauge transformations. The divergence of j'^{μ}_A is

$$\begin{aligned}\partial_\mu j'^{\mu}_A(x) = & -\frac{ig}{2}\lim_{\epsilon\to 0}\epsilon^\nu\Bigg[F_{\mu\nu}(x) * \psi^t\left(x+\frac{\epsilon}{2}\right)(\gamma^\mu\gamma^5)^t * \bar{\psi}^t\left(x-\frac{\epsilon}{2}\right) + \\ & + \psi^t\left(x+\frac{\epsilon}{2}\right)(\gamma^\mu\gamma^5)^t * \bar{\psi}^t\left(x-\frac{\epsilon}{2}\right) * F_{\mu\nu}(x)\Bigg]. \end{aligned} \tag{4.7}$$

It is by now well known that the current j'_A satisfies the non-commutative counterpart of the usual anomaly equation of gauge theories [7, 8]. We can easily see how this result arises in the point splitting approach. The crucial difference between the equations for the divergence of j_A and j'_A is the order in which the field strength and the fermionic fields are multiplied. In (4.6) the field strength does not separate ψ and $\bar{\psi}$ and the effective regularization of the current due to the star-product does not take place. Namely, the divergence of j'_A is given by equation (4.5) with the argument of the l-integration being just ϵ instead of $\epsilon - \theta q$.

We will evaluate now the divergence of the non-planar current j_A in four dimensions using the same approach. The main difference with the two-dimensional case is that the relevant contributions to the fermion correlator in (4.4) come from first and second order in perturbation theory. The contribution from tree level cancels since $\mathrm{tr}\,(\gamma^\rho\gamma^\mu\gamma^5) = 0$ in $d>2$. At first order in perturbation theory we have

$$-ig\left\langle \bar{\psi}(l)\gamma^\mu\gamma^5\psi(p)\int d^4x\,\bar{\psi} * \not{A} * \psi\right\rangle = -4g\epsilon^{\mu\rho\alpha\beta}\frac{l_\rho p_\alpha}{l^2p^2}A_\beta(p+l)\,e^{\frac{i}{2}p\theta l}, \tag{4.8}$$

where we have used that in four dimension $\mathrm{tr}\,(\gamma^\mu\gamma^\rho\gamma^\alpha\gamma^\beta\gamma^5) = -4\,i\,\epsilon^{\mu\rho\alpha\beta}$. Defining $k=p+l$ and substituting the previous equation in (4.4), we obtain

$$\begin{aligned} iq_\mu j^\mu_A(q) &= 4ig^2\lim_{\epsilon\to 0}\epsilon^\nu\epsilon^{\mu\rho\alpha\beta}\int\frac{d^4k}{(2\pi)^4}\frac{d^4l}{(2\pi)^4}\frac{l_\rho e^{il(\epsilon-\theta q)}}{l^2(k-l)^2}F_{\mu\nu}(q-k)k_\alpha A_\beta(k)e^{-\frac{i}{2}q\theta k} \\ &= -\frac{g^2}{2\pi^2}\lim_{\epsilon\to 0}\frac{\epsilon^\nu(\epsilon-\theta q)_\rho}{(\epsilon-\theta q)^2}\epsilon^{\mu\rho\alpha\beta}\int\frac{d^4k}{(2\pi)^4}f(k,|\epsilon-\theta q|)F_{\mu\nu}(q-k)ik_\alpha A_\beta(k)e^{-\frac{i}{2}q\theta k}. \end{aligned} \tag{4.9}$$

The l-integration in this case produces, in addition to $(\epsilon-\theta q)_\rho/(\epsilon-\theta q)^2$, the function $f(k,|\epsilon-\theta q|)$. f can be expressed in terms of Bessel functions, it is always finite and tends to 1 as $|\epsilon-\theta q|$ tends to zero. Thus the r.h.s. of (4.9) vanishes for $\theta q \neq 0$, as it happens in the two-dimensional case. For $\theta q = 0$ it gives clearly a non-zero result. It is straightforward to analyze the contribution to the anomaly equation from evaluating the fermion correlator at second order in perturbation theory. We will not do it explicitly, since the pattern is as before. At $\theta q \neq 0$ the anomaly cancels. The result at $\theta q = 0$ is non-zero, combining with (4.9) to promote $i(k_\alpha A_\beta(k) - k_\beta A_\alpha(k)) \to F_{\alpha\beta}(k)$. Restricted to $\theta q = 0$, (4.9) reduces to

$$iq_\mu j^\mu_A(q) = -\frac{g^2}{16\pi^2}\int \frac{d^4k}{(2\pi)^4}\,\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}(q-k)F_{\alpha\beta}(k)\,, \tag{4.10}$$

which reproduces the integrated anomaly equation (2.6).

5. Mixed anomalies

Similar arguments apply to the local anomalies of chiral gauge theories. Consider a simple case among those treated in [10], a $\mathrm{U}(1)\times\mathrm{U}(1)$ gauge theory with a chiral fermion in the bifundamental representation. In the case of a right handed fermion, the vector currents associated to the groups $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ are

$$j^\mu_1 = \psi^t(\gamma^\mu P_+)^t * \bar\psi^t\,, \qquad j^\mu_2 = \bar\psi * (\gamma^\mu P_+)\psi\,, \tag{5.1}$$

with $P_+ = \frac{1}{2}(1+\gamma^5)$. These two currents are in one to one correspondence with those in (2.3). Thus the analysis of previous sections applies straightforwardly to local mixed anomalies. We will reduce now to the four dimensional case and only spatial non-commutativity. Equation (2.6) implies the existence of a mixed $\mathrm{U}(1)_i\,\mathrm{U}(1)^2_j$ anomaly restricted to $\theta q = 0$. Suppose that we add the necessary chiral matter to cancel $\mathrm{U}(1)^3$ anomalies, leaving the mixes anomalies untouched. Then we have

$$\int dx_{NC}\,\partial_\mu j^\mu_{(i)} = -\frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\int dx_{NC}\,F^{(j)}_{\mu\nu}\,F^{(j)}_{\rho\sigma}\,, \tag{5.2}$$

for $i \neq j$. The variation of the action induced by a gauge transformation $\lambda^{(i)}$ due to the anomaly (5.2) is given in Fourier space by

$$\delta_{\lambda^{(i)}}S = \frac{1}{16\pi^2}\int d^4q\,\lambda^{(i)}(q)F\tilde F^{(j)}(-q)|_{\theta q=0}\,. \tag{5.3}$$

The variation (5.3) vanishes unless the gauge parameter is a delta function $\lambda(q) \propto \delta(\theta q)$. In coordinate space the action (5.3) takes the form

$$\delta_{\lambda^{(i)}}S = \frac{1}{16\pi^2 V_{NC}}\int d^2x\left(\int d^2x_{NC}\,\lambda^{(i)}\right)\left(\int d^2x_{NC}\,F\tilde F^{(j)}\right), \tag{5.4}$$

where V_{NC} represents the (infinite) volume of the non-commutative directions, and x and x_{NC} denote respectively the ordinary and non-commutative directions. The variation of the action is non-zero only if the integration of λ along the non-commutative directions would cancel the volume factor in the denominator of (5.4).

6. The noncommutative Green-Schwarz mechanism

In this section we address the paradox of the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold, which was mentioned in the introduction and which was raised in [14]. Consider a collection of N D3 branes, placed on a $\mathbb{C}^3/\mathbb{Z}_3$ orbifold singularity in a background of a constant NS-NS 2 form. The resulting noncommutative field theory is a $\mathrm{U}(N)^3$ field theory with three generations of bi-fundamental chiral multiplets. The matter spectrum is summarized in table 1.

JHEP06(2002)050

Since the theory is chiral there are potential anomalies. The $U_i(N)^3$ anomalies cancel, since there are $3N$ fundamental and $3N$ anti-fundamental fermions charged under each of the gauge groups factors. There is, however, a potential $U_i(N)^2U_j(1)$ anomaly. For example, the first row in table 1 describes the matter content which is charged under both $U_1(N)$ and $U_2(N)$. Only these fermions circulate in a triangle diagram with two external legs in $U_1(N)$ and one leg in $U_2(N)$. One can choose a regularization scheme in which the local $U_2(1)$ gauge symmetry is anomalous. Hence, the field theory is sick (to be more precise, only two combinations out of the three U(1)'s are anomalous). Since this theory arises from a string theory setup, it should be consistent — probably even in the decoupling limit $\alpha' \to 0$

	$U_1(N)\times$	$U_2(N)\times$	$U_3(N)$
3 chirals	$\square$	$\overline{\square}$	1
3 chirals	1	$\square$	$\overline{\square}$
3 chirals	$\overline{\square}$	1	$\square$

Table 1: The matter content of the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold theory.

Let us review first the string theory solution to this problem in the commutative case [30]. The type-IIB closed string spectrum contains two massless 0-forms fields in the twisted sectors which are localized at the orbifold singularity. Their coupling to the brane can be written in a convenient way in terms of three fields $C^{(i)}$. They transform under gauge transformations $\delta(\mathrm{tr}\, A_\mu^{(i)}) = \partial_\mu \epsilon^{(i)}$ and $\delta C^{(i)} = -\epsilon^{(i)}$. The $C^{(i)}$ are constraint to sum to zero in a gauge invariant way. The action is

$$S \sim \int d^4x \Bigg\{ \frac{1}{\alpha'} \sum_{i=1}^{3} \Big((\mathrm{tr}\, A_\mu^{(i)} + \partial_\mu C^{(i)})^2 + \lambda(\partial^2 C^{(i)} + \mathrm{tr}\, \partial A^{(i)}) \Big) + \\ + \Big(C^{(1)}\, (\mathrm{tr}\, F_{\mu\nu}^{(2)} \tilde{F}_{\mu\nu}^{(2)} - \mathrm{tr}\, F_{\mu\nu}^{(3)} \tilde{F}_{\mu\nu}^{(3)}) + \text{cyclic perm.} \Big) \Bigg\}. \tag{6.1}$$

Upon integration over the three twisted RR-fields and the Lagrange multiplier λ we get the effective action

$$S_{\text{eff}} = \frac{3}{16\pi^2} \int d^4x \left\{ (\mathrm{tr}\, \partial \cdot A^{(1)}) \frac{1}{\partial^2} \left(\mathrm{tr}\, F\tilde{F}^{(2)} - \mathrm{tr}\, F\tilde{F}^{(3)} \right) + \text{cyclic perm.} \right\}. \tag{6.2}$$

In addition there is a rank two mass matrix for the three $A_\mu^{(i)}$'s and also $(F\tilde{F})^2$ terms. The full partition function is gauge invariant. Under a $U_i(1)$ gauge rotation, the anomaly is compensated by the gauge transformation of (6.2). In a diagrammatic language it is described by figure 3. The triangle diagram (figure 3*a*) is cancelled by a tree level diagram which involves an exchange of closed RR field (figure 3*b*), in a generalized Green-Schwarz mechanism. The diagram, figure 3*b*, can be understood as due to an annulus diagram, with two U(N) insertions on one boundary and one U(1) insertion at the second boundary (see figure 4). Thus, in the commutative case gauge invariance is restored by an axion like field (twisted RR field) that cancels the gauge anomaly and gives a gauge invariant mass $M^2 \sim 1/\alpha'$ to the two anomalous U(1)'s. The third U(1) remains massless. The low energy theory, in the commutative case, would therefore be an SU$(N)^3 \times$ U(1) gauge theory. The two apparently anomalous local U(1)'s became global and decoupled, yielding a consistent low-energy theory.

JHEP06(2002)050

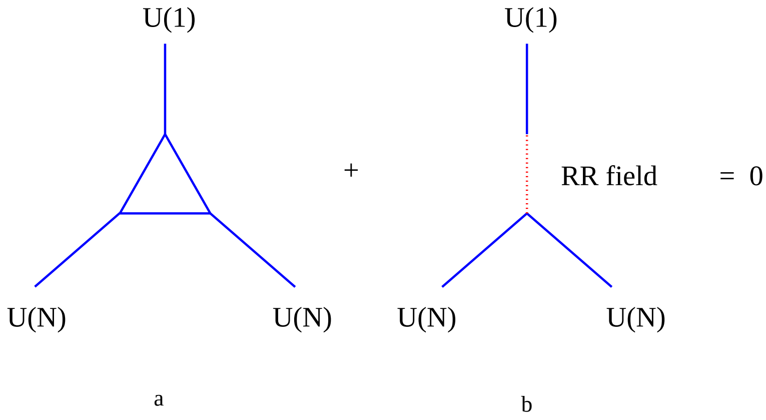

Figure 3: The Green-Schwarz mechanism.

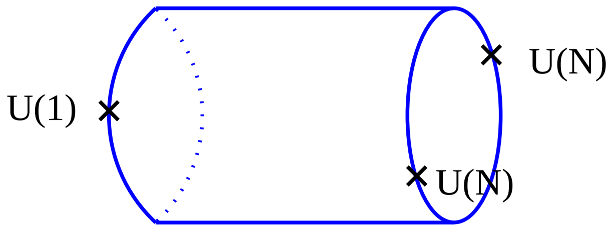

Figure 4: The annulus diagram with $\mathrm{U}(N)^2$ insertions on one boundary and $\mathrm{U}(1)$ insertion at the other.

Let us now return to the more complicated non-commutative case. We will consider again the case where the non-commutativity affects only (two) space directions. Notice that the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold gauge theory has $\mathcal{N}=1$ supersymmetry. Thus it is free from the dangerous quadratic infrared divergences due to UV/IR mixing effects, which could render the theory unstable [31, 32, 33]. However, we would like to stress that the triangle anomaly does not vanish, due to the contribution from zero non-commutative momentum flowing into the triangle diagram at the non-planar vertex. This is exactly the non-planar anomaly which was described in the previous sections. Moreover, the problem is more severe in the noncommutative case, since U(1) couples to the rest of the $\mathrm{SU}(N)$ theory due to non-commutative gauge invariance [23] and therefore the problem cannot be solved by making the U(1) global (massive).

Furthermore, the action (6.1) now presents some immediate problems. We will assume that if a Green-Schwarz mechanism exists in the non-commutative case, it must be possible to formulate it naturally in terms of the non-commutative gauge field A_μ. The reason for this is that the non-planar anomaly has a simple expression in terms of A_μ, instead of involving arbitrary powers of this field. However, the first term in (6.1) would not be gauge invariant under non-commutative gauge transformations. Second the non-planar anomaly, diagram 3a, exits only for zero non-commutative momentum $\theta q = 0$. However the diagram 3b mediated by the action (6.1) seems to exist also for non-zero θq.

In spite of these problems, it is easy to propose an action similar to (6.2) whose variation under gauge transformations would cancel the non-planar anomaly

$$S_{\text{eff}} = \frac{3}{16\pi^2 V_{NC}} \int d^2x \left\{ \left(\mathrm{tr}\, \partial \cdot A^{(1)}\right)_0 \frac{1}{\partial^2} \left(\mathrm{tr}\, F\tilde{F}^{(2)} - \mathrm{tr}\, F\tilde{F}^{(3)}\right)_0 + \text{cyclic perm.} \right\}, \quad (6.3)$$

where, as in section 5, V_{NC} is the volume of and the subscript "$_0$" denotes integration along the non-commutative plane. The coordinates x as well as the derivatives in (6.3) are restricted to the two commutative directions. It is immediate that the gauge variation of the previous expression can cancel the variation of the action due to the non-planar anomaly, eq. (5.4). The action (6.3) is well defined under *noncommutative* gauge transformation since each trace operator is separately integrated along the non-commutative plane. If we interpret (6.3) as derived from a diagram as in figure 3*b*, it would correspond to restricting the closed string exchange to $\theta q = 0$. Notice that only RR fields from the twisted sectors can participate in the Green-Schwarz mechanism. In the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold there are only two twisted massless 0-form RR fields, which should couple to a combination of U(1)'s as in the ordinary case. Although a sum of two U(1)'s is *not* a noncommutative U(1), one is allowed to consider linear combinations at $\theta q = 0$, where the non-commutative structure trivializes.

We now indicate how the action (6.3) results from string theory. Consider the annulus diagram, figure 4. The calculation of this diagram for the noncommutative theory, based on the expression for the ordinary theory [34], was carried out in [14]

$$A^{\mu\nu} \propto \epsilon^{\mu\nu\rho\sigma} p_\rho k_\sigma \lim_{M\to\infty} M^2 \int_0^\infty dt\; e^{-M^2 t} e^{-\frac{\pi}{2t}(\theta q)^2} \int_0^1 d\nu_{1,2} e^{-\frac{i}{2} p\theta k(2\nu_{12}+\epsilon(\nu_{12}))}\,, \tag{6.4}$$

where M is a regulator, ϵ is the step function and $\nu_{12} = \nu_1 - \nu_2$. For any non-zero θq the integral near $t = 0$ (the UV) is exponentially suppressed and yields a zero anomaly. However, for $\theta q = 0$ this is no longer true and we have an anomaly. The interpretation is that for $\theta q = 0$ we have an anomaly in field theory (non-vanishing triangle diagram) and it is compensated in a Green-Schwarz mechanism by an exchange of a massless RR field. The mechanism for $\theta q = 0$ is exactly the same as in the commutative theory.

For $\theta q \neq 0$ the scenario is different. The amplitude (6.4) vanishes in this case, indicating that there is no anomaly in the field theory. The crucial difference with respect to the previous case is that, in considering a closed string exchange as in figure 3*b*, we cannot discard the massive string modes [35]. The on-shell condition for twisted closed string modes is $g^{\mu\nu} q_\mu q_\nu + N/\alpha' = 0$, with g denoting the closed string metric. The open and closed string metrics are related via [36]

$$g^{-1} = G^{-1} - \frac{\theta G \theta}{(2\pi\alpha')^2}, \tag{6.5}$$

and thus the on-shell condition is

$$q^2 + \frac{(\theta q)^2}{(2\pi\alpha')^2} = -\frac{N}{\alpha'}\,. \tag{6.6}$$

In the $\alpha' \to 0$ limit, the oscillator mass becomes a subleading effect with respect to the momentum along the non-commutative directions. Assuming that the massive RR string modes have similar couplings to the field theory operators as the massless one, it is clear that all of them will contribute to diagram 3b when $\theta q \neq 0$. Such an exchange is then more adequately interpreted directly in field theory terms. Its effect is to introduce the non-commutative damping factor $e^{-\frac{\pi}{2t}(\theta q)^2}$ that leads to the cancellation of (6.4). Notice that in the $\alpha' \to 0$ limit, modes with $\theta q \neq 0$ are never on-shell because the kinetic term is suppressed by two powers of α' with respect to the momentum in the non-commutative directions. Contrary, when $\theta q = 0$ the oscillator mass is the dominant effect in (6.6). This selects the massless closed string mode, bringing us back to the ordinary Green-Schwarz mechanism.

The Green-Schwarz mechanism has its origin in the fact that non-planar string annulus diagrams are generically finite. The absence of divergences in the non-planar annulus can be understood in terms of the closed string modes using channel duality. Consequently, closed string modes play a fundamental role in the Green-Schwarz mechanism [4]. The (partial) cancellation of the non-planar anomaly in non-commutative gauge theories occurs because of the regularization of the associated triangle graph by non-commutative effects. This regularization of otherwise divergent Feymann graphs gives rise to one of the most remarkable characteristics of non-commutative field theories: UV/IR mixing [20]. In [20, 37, 38] the similarity between the non-planar graphs in non-commutative field theories and in string theory was stressed. Following these ideas, [39] showned that the infrared divergences associated with UV/IR mixing effects in gauge theories can be reproduced in terms of closed string exchange, where the whole tower of closed string modes contributes. We propose that a similar pattern applies to the cancellation of non-planar anomalies (see also [40] for a related discussion about flavour anomalies). In this sense, it can be interpreted as an automatic Green-Schwarz mechanism [14].

Since at $\theta q = 0$ we have argued that an ordinary Green-Schwarz mechanism takes place, we propose the following lagrangian for the 'mass term' of the U(1) in the noncommutative theory

$$\mathcal{L} \sim \frac{1}{\alpha'}(\mathrm{tr}\, A_\mu + \partial_\mu C)(q) G^{\mu\nu} (\mathrm{tr}\, A_\nu + \partial_\nu C)(-q)|_{\theta q = 0} \,. \tag{6.7}$$

The lagrangian (6.7) is gauge invariant with respect to noncommutative gauge transformations. Now, we add to the action the 'axion' term $CF\tilde{F}$ and we integrate over the RR field. The resulting action is (6.3).

There is a subtle issue that should be discussed. The WZ couplings in the noncommutative theory are more involved than in the ordinary case, see details at [41, 42]. However, we assume that the existence of a similar piece at zero noncommutative momentum is enough for the anomaly cancellation.

We have argued that the non-commutative gauge theory on the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold has non-vanishing mixed anomalies. However they only affect U(1) modes with zero non-commutative momentum. For these modes both the non-commutative and the non-abelian structure trivialize, i.e. they do not mix with the rest under gauge transformations. In (6.7) we have proposed that due to the interaction with the RR field, the $\theta q = 0$ components of two of the three U(1)'s become massive. A linear combination of U(1)'s remains anomaly free as in the ordinary case. Thus the non-commutative $\mathrm{U}(N)^3$ structure is preserved, since this only requires that the U(1) excitations with non-zero noncommutative momentum remain coupled. The excitations with zero noncommutative momentum become massive and decouple. It is interesting that this splitting, which of course violates Lorentz symmetry, preserves the full gauge symmetry.

JHEP06(2002)050

7. Global anomalies

In this section we address the issue of global anomalies. As we have already mentioned in the introduction, there are two different kinds of anomalies in noncommutative theories: planar and nonplanar. In particular there are two kinds of global anomalies. Since planar global anomalies are as in the commutative situation, we will only consider nonplanar anomalies.

Consider a fermion that transforms in the bifundamental representation of a product group $G \times H$, where H is a global symmetry and G is either local or global. Both cases have interesting applications in particle physics.

The prime example of mixed global and local symmetries is the $\pi^0 \to 2\gamma$ process. In this case the global symmetry is $\mathrm{U}(1)_A$ and the local is $\mathrm{U}(1)$ (Maxwell). According to the analysis in the previous sections the axial charge is not conserved. Thus, though we cannot write a local anomaly equation, we see that no conservation of charge forbids the pion decay, in contrast to [14].

The second important case is when both G and H are global. Here the global anomaly restricts the microscopic structure of the theory (or the description in terms of a dual theory, as in [3]). According to 't Hooft [2], the anomaly, which is nothing but the number of fermions species circulating in the triangle loop, should be the same in the two descriptions (see also [43]). This condition imposes stringent restrictions on a possible dual description.

Had the nonplanar anomaly vanished, this restriction could have been removed. However, we claim that the same anomaly matching conditions as in the commutative theory should be imposed. To see that, we can repeat the reasoning of 't Hooft. One can gauge the global symmetry with a very weak coupling[6] and add chiral matter to cancel the anomaly. The same 'hidden' chiral sector should guarantee an anomaly free gauge theory in the two descriptions. Therefore the global anomaly should be the same. For this kind of reasoning in favor of the anomaly matching conditions it is enough that an integral version of the anomaly exist, since even an integral version of the anomaly renders the local theory inconsistent. Thus, the noncommutative theory is not less restrictive than the commutative one.

Acknowledgments

We would like to thank L. Alvarez-Gaumé, C. Angelantonj, F. Ardalan, O. Bergman, S. Elitzur, A. Font, Y. Frishman, K. Landsteiner, C. Martín, R. Minasian, R. Rabadán, C. Scrucca, S. Silva, M. Vazquez-Mozo, Y. Oz, A. Uranga and F. Zamora for discussions. We would like to thank A. Schwimmer for his comments on the first version of the paper. They led to the additional discussion at the end of section 3.2. S.T. would like to thank the Isaac-Newton-Institute for Mathematical Sciences for hospitality during the final stages of this work. A.A. thanks the Albert-Einstein-Institut for the warm hospitality where part of this work was done. This work was partially supported by GIF, the German-Israeli Foundation for Scientific Research.

[6]We assume here a global symmetry that has a natural realization in terms of a $\mathrm{U}(N)$ group. Also, in order to have any potential local anomaly we must work in more than two dimensions with some commutative directions.

References

[1] S.B. Treiman, E. Witten, R. Jackiw and B. Zumino, *Current algebra and anomalies*, World Scientific, Singapore 1985.

[2] G. 't Hooft, in *Recent developments in gauge theories*, G. 't Hooft, C. Itzykson, A. Jaffe, H. Lehmann, P. K. Mitter, I.M. Singer and R. Stora eds., Plenum Press, New York 1980.

[3] N. Seiberg, *Electric-magnetic duality in supersymmetric nonabelian gauge theories*, *Nucl. Phys.* **B 435** (1995) 129 [hep-th/9411149].

[4] M.B. Green and J.H. Schwarz, *Anomaly cancellation in supersymmetric $D = 10$ gauge theory and superstring theory*, *Phys. Lett.* **B 149** (1984) 117.

[5] M.R. Douglas and N.A. Nekrasov, *Noncommutative field theory*, *Rev. Mod. Phys.* **73** (2002) 977 [hep-th/0106048].

[6] R.J. Szabo, *Quantum field theory on noncommutative spaces*, hep-th/0109162.

[7] F. Ardalan and N. Sadooghi, *Axial anomaly in non-commutative QED on R^4*, *Int. J. Mod. Phys.* **A 16** (2001) 3151 [hep-th/0002143].

[8] J.M. Gracia-Bondia and C.P. Martin, *Chiral gauge anomalies on noncommutative R^4*, *Phys. Lett.* **B 479** (2000) 321 [hep-th/0002171].

[9] L. Bonora, M. Schnabl and A. Tomasiello, *A note on consistent anomalies in noncommutative YM theories*, *Phys. Lett.* **B 485** (2000) 311 [hep-th/0002210].

[10] C.P. Martín, *The UV and IR origin of non-abelian chiral gauge anomalies on noncommutative Minkowski space-time*, *J. Phys.* **A 34** (2001) 9037 [hep-th/0008126].

[11] F. Ardalan and N. Sadooghi, *Anomaly and nonplanar diagrams in noncommutative gauge theories*, *Int. J. Mod. Phys.* **A 17** (2002) 123 [hep-th/0009233].

[12] C.P. Martín, *Chiral gauge anomalies on noncommutative Minkowski space-time*, *Mod. Phys. Lett.* **A 16** (2001) 311 [hep-th/0102066].

[13] J. Nishimura and M.A. Vazquez-Mozo, *Noncommutative chiral gauge theories on the lattice with manifest star-gauge invariance*, *J. High Energy Phys.* **08** (2001) 033 [hep-th/0107110].

[14] K.A. Intriligator and J. Kumar, *$*$-wars episode 1: the phantom anomaly*, *Nucl. Phys.* **B 620** (2002) 315 [hep-th/0107199].

[15] L. Bonora and A. Sorin, *Chiral anomalies in noncommutative YM theories*, *Phys. Lett.* **B 521** (2001) 421 [hep-th/0109204].

[16] C.P. Martin, *The covariant form of the gauge anomaly on noncommutative R^{2N}*, *Nucl. Phys.* **B 623** (2002) 150 [hep-th/0110046].

[17] R. Banerjee and S. Ghosh, *Seiberg Witten map and the axial anomaly in noncommutative field theory*, *Phys. Lett.* **B 533** (2002) 162 [hep-th/0110177].

[18] A. Gonzalez-Arroyo and M. Okawa, *A twisted model for large-N lattice gauge theory*, *Phys. Lett.* **B 120** (1983) 174; *The twisted Eguchi-Kawai model: a reduced model for large-N lattice gauge theory*, *Phys. Rev.* **D 27** (1983) 2397.

[19] T. Filk, *Divergencies in a field theory on quantum space*, *Phys. Lett.* **B 376** (1996) 53.

[20] S. Minwalla, M. Van Raamsdonk and N. Seiberg, *Noncommutative perturbative dynamics*, *J. High Energy Phys.* **02** (2000) 020 [hep-th/9912072].

[21] A. Armoni, R. Minasian and S. Theisen, *On non-commutative $N = 2$ super Yang-Mills*, *Phys. Lett.* **B 513** (2001) 406 [hep-th/0102007].

[22] K. Matsubara, *Restrictions on gauge groups in noncommutative gauge theory*, *Phys. Lett.* **B 482** (2000) 417 [hep-th/0003294].

[23] A. Armoni, *Comments on perturbative dynamics of non-commutative Yang-Mills theory*, *Nucl. Phys.* **B 593** (2001) 229 [hep-th/0005208].

[24] F. Zamora, *On the operator product expansion in noncommutative quantum field theory*, *J. High Energy Phys.* **05** (2000) 002 [hep-th/0004085].

[25] J. Gomis and T. Mehen, *Space-time noncommutative field theories and unitarity*, *Nucl. Phys.* **B 591** (2000) 265 [hep-th/0005129].

[26] K. Landsteiner, E. Lopez and M.H.G. Tytgat, *Excitations in hot non-commutative theories*, *J. High Energy Phys.* **09** (2000) 027 [hep-th/0006210].

[27] A. Hashimoto and N. Itzhaki, *Traveling faster than the speed of light in non-commutative geometry*, *Phys. Rev.* **D 63** (2001) 126004 [hep-th/0012093].

[28] L. Alvarez-Gaumé, J.L.F. Barbón and R. Zwicky, *Remarks on time-space noncommutative field theories*, *J. High Energy Phys.* **05** (2001) 057 [hep-th/0103069].

[29] C.P. Martín and D. Sanchez-Ruiz, *The one-loop UV divergent structure of* U(1) *Yang-Mills theory on noncommutative R^4*, *Phys. Rev. Lett.* **83** (1999) 476 [hep-th/9903077].

[30] L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Anomalous* U(1)*'s in type-I and type-IIB $D = 4$, $N = 1$ string vacua*, *Nucl. Phys.* **B 542** (1999) 112 [hep-th/9808139].

[31] A. Matusis, L. Susskind and N. Toumbas, *The IR/UV connection in the non-commutative gauge theories*, *J. High Energy Phys.* **12** (2000) 002 [hep-th/0002075].

[32] F.R. Ruiz, *Gauge-fixing independence of IR divergences in non-commutative* U(1)*, perturbative tachyonic instabilities and supersymmetry*, *Phys. Lett.* **B 502** (2001) 274 [hep-th/0012171].

[33] K. Landsteiner, E. Lopez and M.H.G. Tytgat, *Instability of non-commutative SYM theories at finite temperature*, *J. High Energy Phys.* **06** (2001) 055 [hep-th/0104133].

[34] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory. Vol. 2: loop amplitudes, anomalies and phenomenology*, Cambridge University Press, Cambridge 1987.

[35] G. Arcioni, J.L.F. Barbón, J. Gomis and M.A. Vazquez-Mozo, *On the stringy nature of winding modes in noncommutative thermal field theories*, *J. High Energy Phys.* **06** (2000) 038 [hep-th/0004080].

[36] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[37] H. Liu and J. Michelson, *$*$-trek: the one-loop $N = 4$ noncommutative SYM action*, *Nucl. Phys.* **B 614** (2001) 279 [hep-th/0008205].

[38] Y. Kiem, S. Lee, S.-J. Rey and H.-T. Sato, *Interacting open wilson lines in noncommutative field theories*, *Phys. Rev.* **D 65** (2002) 046003 [hep-th/0110215]; *Anatomy of two-loop effective action in noncommutative field theories*, hep-th/0110066.

[39] A. Armoni and E. Lopez, *UV/IR mixing via closed strings and tachyonic instabilities*, *Nucl. Phys.* **B 632** (2002) 240 [hep-th/0110113].

JHEP06(2002)050

[40] K. Intriligator and J. Kumar, *The secret gauging of flavor symmetries in noncommutative QFT*, *Nucl. Phys.* **B 624** (2002) 299 [hep-th/0111007].

[41] S. Mukhi and N.V. Suryanarayana, *Chern-Simons terms on noncommutative branes*, *J. High Energy Phys.* **11** (2000) 006 [hep-th/0009101].

[42] Y. Okawa and H. Ooguri, *An exact solution to Seiberg-Witten equation of noncommutative gauge theory*, *Phys. Rev.* **D 64** (2001) 046009 [hep-th/0104036];
S. Mukhi and N.V. Suryanarayana, *Gauge-invariant couplings of noncommutative branes to Ramond-Ramond backgrounds*, *J. High Energy Phys.* **05** (2001) 023 [hep-th/0104045];
H. Liu and J. Michelson, *Ramond-Ramond couplings of noncommutative D-branes*, *Phys. Lett.* **B 518** (2001) 143 [hep-th/0104139].

[43] Y. Frishman, A. Schwimmer, T. Banks and S. Yankielowicz, *The axial anomaly and the bound state spectrum in confining theories*, *Nucl. Phys.* **B 177** (1981) 157.